

An Introduction to Autonomous Dynamics and Its Classification on Compact Surfaces

Kambiz Moghaddamfar

October 10, 2026

Abstract

In this talk, the first part of a two-part series, we introduce the notion of autonomous dynamics and present a classification of autonomous diffeomorphisms on compact surfaces.

Let M be a compact smooth manifold of dimension n endowed with a parallelization F of its tangent bundle, i.e., a system of vector fields defining a basis of each tangent space, and let φ be a C^1 diffeomorphism of M . The derivative cocycle is the map

$$x \in M \mapsto C_\varphi(x) \in GL(n, \mathbb{R}),$$

where $C_\varphi(x)$ is the matrix of $D_x\varphi : T_xM \rightarrow T_{\varphi(x)}M$ with respect to the bases given by $F(x)$ and $F(\varphi(x))$, respectively.

We say that φ is autonomous with respect to F if the cocycle map C_φ is constant. The same definition applies to a C^1 action of a group G if every element is autonomous with respect to F . In the autonomous case, if $C_\varphi(x) \equiv C$, then

$$C_{\varphi^k}(x) = C^k$$

for every $x \in M$. Thus, the Lyapunov exponents are independent of x and, in particular, constant on M .

This rigidity condition creates a connection between geometric structures, algebraic models, and dynamical behavior. In dimension two, we will focus on the classification of autonomous diffeomorphisms on compact surfaces. The existence of a global framing forces the surface to be a torus, and the dynamics can then be understood through the associated cocycle matrix. In dimension two, the hyperbolic, elliptic, and parabolic cases correspond, respectively, to linear Anosov automorphisms, isometries, and affine models.